Potentially large uncertainty in ecosystem carbon dynamics resulting from ambiguous numerical coupling of carbon and nitrogen biogeochemistry: A demonstration with the ACME land model

Jinyun Tang, William J. Riley
Earth & Environmental Sciences Area, Lawrence Berkeley National Laboratory, Berkeley, 94720, USA

Correspondence to: Jinyun Tang (jinyuntang@lbl.gov)

Abstract. Most Earth System Models (ESMs) have incorporated, or are incorporating, coupled carbon and nutrient dynamics in their land modules. We show here that different numerical implementations of nutrient controls may imply different ecological mechanisms not recognized in the original model design and can have first order impacts on predicted terrestrial carbon cycling. Using the version-zero land module of the DOE ESM ACME (ALMv0), we analysed land-atmosphere CO₂ exchange with coupled carbon and nitrogen dynamics through three commonly-applied numerical implementations of the supply-demand ansatz based nitrogen limitation: (1) Mineral Nitrogen based Limitation (MNL), (2) Net nitrogen Uptake based Limitation (NUL), and (3) Proportional Nitrogen flux based Limitation (PNL). By the last decade of the contemporary period (1850-2000), the three schemes resulted in very similar global terrestrial carbon and nitrogen distributions. However, under the RCP4.5 CO₂ concentration forcing, these implementations resulted in wildly diverging 2001-2300 land-atmosphere CO₂ exchanges. Quantitatively, the divergence is as large as that of the CMIP5 models by 2100 and is about 1900 Pg C (~890 ppmv) by 2300. Our analysis suggests that these differences result from: (1) the typically predicted high terrestrial ecosystem carbon to nitrogen ratios (i.e., nutrient constrained conditions), (2) the schemes predict different levels of limitation to the nitrogen-regulated processes, so that the PNL scheme favours larger nitrogen loss through aerobic and anaerobic denitrification and surface and subsurface hydrological transport and (3) the highly sensitive response of coarse woody debris dynamics to nitrogen limitation. We also found significant sensitivity of model predictions to initial conditions and numerical time step size but insignificant sensitivity to the sequence of numerical oxygen and nitrogen limitation or the ordering of calculation for reaction and chemical transport. We conclude that inconsistencies in numerical impositions of nutrient limitations have the potential to produce large uncertainties in predicted carbon stocks and long-term carbon-climate feedbacks. Finally, we recommend approaches to systematically alleviate these uncertainties.

Keywords: carbon-nitrogen feedbacks, nitrogen limitation, land-atmosphere CO₂ exchange, RCP4.5, law of the minimum
1 Introduction

Earth System Models (ESMs) used for assessing future climate and related processes rely on large-scale land biogeochemical (BGC) models to simulate ecosystem responses to changing atmospheric CO₂, temperature, precipitation, nitrogen (N) deposition, and etc. Recent work analysing ESM land models that participated in the Coupled Model Intercomparison Project Phase 5 (CMIP5) showed very large differences among those models’ predictions (e.g., Arora et al., 2013; Friedlingstein et al., 2014; Shao et al., 2013; Koven et al. 2015). Such differences are often attributed to the four types of uncertainties, including structural (Tang et al., 2010; Wieder et al., 2015), numerical (Yeh and Tripathi, 1989), parameterization (Tang and Zhuang, 2008; Luo et al., 2015), and forcing data (Clein et al., 2007; Blanke et al., 2016), which are, respectively, loosely related to the four stages of BGC model design: (I) conceptualizing the relevant mechanisms and translating them into governing equations; (II) numerical encoding of the governing equations; (III) process module calibration and parameterization; and (IV) model analyses and applications. There have been numerous examples of how one could quantify and reduce these uncertainties (e.g., Tang and Zhuang, 2008, 2009; Williams et al., 2009; Lichstein et al., 2014; Wei et al., 2014; Shi et al., 2015). Here we describe a new type of uncertainty that is a combination of type-I and type-II, and can result in predictions of ecosystem carbon dynamics as divergent as that of CMIP5 land models.

We report our findings using the carbon-nitrogen coupling as an example; however, this new type of uncertainty is related to the broader issue of carbon-nutrient coupling in all kinds of BGC models. Specifically, it relates to how one should numerically represent the fact that different substrates can limit ecosystem biogeochemical processes under different conditions. For instance, it is believed that many terrestrial ecosystems are nitrogen limited (Vitousek and Howarth, 1991; LeBauer and Treseder, 2008), because breaking down the triple bond of dinitrogen (N₂) and converting it into assimilable forms requires a significant fraction of newly assimilated or reserved carbon (Gutschick, 1987). Tropical forest ecosystems are often regarded as phosphorus limited because of their highly weathered soils (Walker and Syers, 1976), but nitrogen or even carbon or potassium limitation can still occur (e.g., Wright et al., 2011; Fanin et al., 2015). In moist environments, such as wetlands, where organic matter decomposition is more likely oxygen limited, anaerobic decomposition dominates but aerobic decomposition may proceed simultaneously (DeBusk et al., 2001). Given this wide range of substrate limitation conditions, it is therefore logical to ask: how would different numerical treatments of substrate limitation influence the prediction of a land BGC model?

We answer the above question by focusing on nitrogen—the most important macronutrient related to whether or not terrestrial ecosystems could continue to sequester anthropogenic CO₂ (Oren et al., 2001; Drake et al., 2011; Grant, 2013). Following Kovarova-Kova and Eggli (1998)'s use of terms “growth-controlling” and “growth-limiting” in substrate dynamics, we note there are two aspects that determine the modelled influence of nitrogen on ecosystem carbon dynamics: (1) the mechanistic formulation of carbon and nitrogen coupling (i.e. growth-controlling) and (2) the numerical implementation of a given formulation (i.e. growth-limiting). The first aspect regards the analytical formulation of how one or more nutrients mechanistically limit the growth of an organism or a compartment of an organism. We acknowledge that
there has been no consensus on how this should be done, and many opinions are currently under debate (e.g., Kooijman, 1998; Patra, 2011; Agren et al., 2012; Niu et al., 2016; Stocker et al., 2016; Zhu et al., 2016). Therefore, we will present our opinion on this first aspect elsewhere, although some of that has been alluded in Tang and Riley (2013), Zhu and Riley (2015) and Zhu et al. (2016). The second aspect concerns how the coupling between different components as given in an analytical formulation should be achieved in a numerically consistent manner. This second aspect has been rarely discussed in the field of land biogeochemical modelling; event though similar issues (called as multi-physics coupling) have been scrutinized in other branches of earth system modelling. A few excellent examples are Phillips (1956), Arakawa (1965) and Wan et al. (2016) for atmospheric physics, Sandu (2001), Nguyen et al. (2009) and Wan et al. (2013) for atmospheric chemistry, Tang et al. (2015) for soil-plant hydrology and Broekhuizen et al. (2008) for marine biogeochemistry. In a nutshell, all these studies indicate that an inappropriate numerical implementation could render an analytically well-formulated model to behave unrealistically, and calibrating and applying such models (in terms of doing steps III and IV as identified above) will be a waste of resources (as implied in the Lax equivalence theorem (Lax and Richtmyer, 1956)).

For this study, we begin our analysis with the following equation for a generic substrate $S$ in a soil control volume:

$$\frac{dS}{dt} = F_{S,\text{input}} - F_{S,\text{uptake}} \tag{1}$$

where $F_{S,\text{input}}$ and $F_{S,\text{uptake}}$ are, respectively, substrate input (from all sources) and substrate uptake (by all competing entities). Here and below, unless otherwise stated explicitly, we assume the units of all variables in a given equation are consistently defined. To simplify the discussion, we have solved the overall spatiotemporal evolution of substrate $S$ (which is a function of both transport and biogeochemistry) using the operator splitting approach (e.g., Strang, 1968; Tang et al., 2013), so that $F_{S,\text{input}}$ and $F_{S,\text{uptake}}$ in equation (1) only refer, respectively, to substrate release and uptake from the interacting agents. As such, for the substrate $S$ (i.e. mineral nitrogen) that we are interested in here, we henceforth use $S$ and mineral nitrogen interchangeably unless a clarification is required), input $F_{S,\text{input}}$ is microbial nitrogen mineralization from soil organic matter (SOM) decomposition; while $F_{S,\text{uptake}}$ includes plant nitrogen assimilation (to support growth) and microbial nitrogen utilization (to support decomposition, nitrification and denitrification). If the interaction between soil mineral surfaces and ammonium nitrogen is considered (e.g. Gerber et al., 2010), $F_{S,\text{input}}$ and $F_{S,\text{uptake}}$ should be modified accordingly, depending on whether ammonium is adsorbed or desorbed from soil minerals. With the operator splitting approach, nitrogen input from other sources (including fertilization, atmospheric nitrogen deposition, nitrogen fixation) and losses through hydrological transport are integrated separately from the competitive coupling between nitrogen mineralization and assimilation. We have tested this treatment by switching the order between solving the biogeochemical processes and transport and found the ordering affected the results marginally small.
When solved with the forward Euler scheme (e.g., Atkinson, 1989), Equation (1) may be approximated as:

\[ S(t + \Delta t) = S(t) + \left(F_{\text{input}} - F_{\text{uptake}}\right) \Delta t \]  

(2)

With a given numerical time step \( \Delta t \), if \( S(t + \Delta t) \) becomes negative (before any adjustment to the rates that change \( S(t) \)), the biogeochemical system is defined as substrate-\( S \) limited during that numerical time step. Here we once again remind readers not to confuse this definition (of growth limiting substrate) with using different analytical formulations of how nutrients could limit or co-limit the biogeochemical system (i.e., growth-controlling substrates), because this numerical limitation (i.e., growth limiting substrate) will always occur for whatever analytical formulation (of growth controlling substrates) being used. We also note that this numerical definition of nitrogen limitation (which operates on time scales from minutes to hours) appears different from the ecological definition, which is defined as stimulated ecosystem productivity in response to nitrogen addition and operates on time scales from days to years (Vitousek and Howarth, 1991). However, in a BGC model, ecological nitrogen limitation is realized as an emergent response accumulated from many within time-step nitrogen limitations (and should be considered as a combination of growth-controlling and growth-limiting processes).

Using a higher order numerical scheme will not avoid this numerical substrate limitation, and, when substrate limitation occurs, the high order scheme will usually become first order (Bolley and Crouzeix, 1978), a result that also holds for implicit schemes (Hundsdorfer and Verwer, 2003). Higher order accuracy may be achieved if both the substrate production and destruction rates are modified simultaneously (e.g., Burchard et al., 2003), but such an approach will fail when substrate production is independent of consumption, a situation that occurs exactly in the CENTURY-like soil biogeochemical models (Parton et al. 1988; Koven et al. 2013), where, because the different soil organic matter pools are decayed in a linearly dependent manner, the activity of nitrogen mineralizers is independent from that of nitrogen immobilizers (Tang and Riley, 2016). Nor will an adaptive time stepping approach resolve this numerical substrate limitation problem, because in many cases it would require an impractically small time-step to avoid negative numerical solutions (Formaggio and Scott, 2011). Nevertheless, a numerical nitrogen limitation as applied in equation (2) does depend on the time step size, but as we demonstrate below, this uncertainty is secondary to that from using different numerical implementations of the supply-demand ansatz based nitrogen limitation.

We now analyse three legitimate and commonly applied numerical methods to resolve substrate limitation when solving equation (2). We reveal that the three numerical approaches imply different coupling between nitrogen competitors and producers in the model, therefore lead to different (sometimes unacknowledged) ecological coupling between carbon and nitrogen dynamics.

The first nitrogen uptake limitation approach has been adopted by models like CLM-CNP (Yang et al., 2014), BiomeBGC (Thornton et al., 2002), BiomeBGC MuSo (Hidy and Barcza, 2014), JSBACH-CN (Parida, 2011; with
denitrification excluded from \( F_{\text{s,uptake}} \), CLM4.0 (Oleson et al., 2010), CLM4.5 (Oleson et al., 2013), and one version of ALMv1 (the land model in the DOE earth system model ACME-v1). Mathematically, it reads

\[
F_{\text{s,uptake}} = \min \left\{ \frac{S(t) / \Delta t}{F_{\text{s,uptake}}}, 1 \right\} F_{\text{s,uptake}}
\]  

(3)

Equation (3) assumes that the actual total nitrogen uptake \( F_{\text{s,uptake}} \) is limited solely by the available mineral nitrogen \( S(t) \) and is not affected by mineral nitrogen released from SOM decomposition during the numerical time step, which is certainly inconsistent with the governing equation. In the following, we name this approach (i.e., equation (3)) as the Mineral Nitrogen based Limitation scheme (MNL).

In some models, like CABLE (Wang et al., 2010) or the Generic Decomposition and Yield model (Comins and McMurtrie, 1993), the \( j \)-th sub-component \( F_{\text{s,uptake,j}} \) of \( F_{\text{s,uptake}} \) may already include substrate limitation based on the availability of \( S(t) \). These models apply either equation (3) or its variants (to be introduced later), or a “numerical” Monod term (e.g., Tang et al., 2016) that has no chemical or biological kinetic meaning (as in contrast to the enzymatic Monod function) to the \( j \)-th potential uptake flux \( F_{\text{s,uptake,j}} \). When both nitrogen and phosphorus are considered for an entity of fixed stoichiometry (e.g., a decomposing organic matter pool or a microbe), the imposition of substrate limitation is even more uncertain. One approach is to use the potential nitrogen uptake flux \( F_{\text{s,N,uptake}} \) and phosphorus uptake flux \( F_{\text{s,P,uptake}} \) to first calculate the nitrogen-limiting factor \( x_N \) and phosphorus-limiting factor \( x_P \)

\[
x_N = \min \left\{ \frac{\text{MIN}_N / \Delta t}{F_{\text{s,N,uptake}}}, 1 \right\}
\]  

(4)

\[
x_P = \min \left\{ \frac{\text{MIN}_P / \Delta t}{F_{\text{s,P,uptake}}}, 1 \right\}
\]  

(5)

Then Liebig’s law of the minimum is applied by taking the minimum of \( x_N \) and \( x_P \) to compute an overall limiting factor \( x_{\text{lim}} \) that constrains the overall decomposition flux, which by stoichiometry balance will lead to down-regulated nitrogen and phosphorus uptake rates \( F_{\text{s,N,uptake,j}} \) and \( F_{\text{s,P,uptake,j}} \) that are then used to resolve the nitrogen and phosphorus competition. Occasionally, \( x_N \) and \( x_P \) may be calculated using Monod functions for each of the substrate competing entities, leading to
a premature application of the law of the minimum onto the individuals (e.g., Leon and Tumpson, 1975; Danger et al., 2008). However (as we explained in Appendix A), such an application of the ‘law of the minimum’ mistakes the system-wise nutrient limitation as a local constraint on subcomponents (or individuals) of the system. Given the limited amount of nitrogen and phosphorus available for competition, an additional application of either equation (3) (or the to be introduced equations (6) or (7)) may still be imposed to avoid negative numerical solutions when all competing fluxes are resolved (i.e., a second application of the law of the minimum will be introduced automatically through the numerical integration). Such a strategy then leads to a double counting of nutrient limitation if the mass balance is imposed strictly (we note such double counting is different from that discussed in Downing et al. (1999) for phytoplankton fertilization experiment), and if the mass balance is not imposed strictly, an unwanted numerical nutrient fertilization might occur (e.g., the ODE45 solver as demonstrated in Tang and Riley (2016)).

The second nitrogen limitation scheme that we analyse here is represented as:

\[
F_{\text{uptake}} = \min \left\{ \frac{S(t)/\Delta t}{F_{\text{uptake}} - F_{\text{input}}}, 1 \right\} F_{\text{uptake}}
\]

We name equation (6) the Net nitrogen Uptake based Limitation (NUL) scheme (note when NUL is applied, it holds that \(F_{\text{uptake}} - F_{\text{input}} > 0\)). The NUL scheme is based on the approach of derivative clipping, and is used in MATLAB’s ODE45 (Shampine et al., 2005). However, ODE45 imposes equation (6) by violating the law of mass balance (Tang and Riley, 2016). We avoid this problem here by applying the flux adjustment only to \(F_{\text{uptake}}\), because \(F_{\text{input}}\) (as it appears in ALMv0) is assumed independent from substrate \(S\) in equation (6).

The third nitrogen limitation scheme is

\[
\bar{F}_{\text{uptake}} = \min \left\{ \frac{F_{\text{input}} + S(t)/\Delta t}{F_{\text{uptake}}}, 1 \right\} F_{\text{uptake}}
\]

We name equation (7) as the Proportional Nitrogen flux based Limitation (PNL), and it is the only numerical scheme (among the three we analysed) that is consistent with the governing equation (which can be verified by entering equation (7) into equation (2) and observing as required that \(S(t + \Delta t) = 0\) when \(S(t)\) is limiting). PNL assumes that the newly (i.e., within the time step) released mineral nitrogen \(F_{\text{input}}\) and existing (i.e., at the beginning of the time step) mineral nitrogen \(S(t)\) are equally accessible to immobilizers. This assumption is an oversimplification because diffusion can limit the newly released and existing mineral nitrogen from mixing completely in the soil over the typically short time steps in land models (0.5 – 1 h) (Schimel and Bennett, 2004). Therefore PNL will underestimate the true nitrogen limitation. We also note that
when diffusion limitation is ignored, assuming whether or not plants and microbes have absolute priority of newly released mineral nitrogen over existing mineral nitrogen will not change the form of equation (7). A modified PNL scheme that includes diffusion constraints is used in the *ecosys* model (e.g. Grant, 2013) to rectify overly large nutrient uptake fluxes (personal discussion with R. Grant, 2016) that can lead to negative nutrient concentrations.

There have been other *numerical* schemes proposed for nitrogen limitation (which however will not be analysed in this study). For instance, Wang et al. (2010) in their constraint of decomposition due to nitrogen limitation (cf. their equation C12) calculated the de facto decomposer nitrogen uptake as

\[
F = \min \left\{ \max \left\{ 1 + \frac{F_{s,\text{input}} - F_{s,\text{uptake}}}{s(t)} \Delta t \right\}, 1 \right\} F_{s,\text{uptake}}
\]

(8)

where \(F_{s,\text{input}}\) and \(F_{s,\text{uptake}}\) refer, respectively, to nitrogen mineralization and microbial nitrogen immobilization. Equation (8) reduces nitrogen uptake when the net mineralization \(F_{s,\text{input}} - F_{s,\text{uptake}}\) is negative, and has no effect on nitrogen uptake when net mineralization is positive (we acknowledge that equation (8) is a more complete form with respect to their equation C12 because their equation C12 was only applied to negative net nitrogen mineralization). But as we explained above, this approach will not avoid predicted negative nitrogen concentrations and further adjustments as represented in the MNL, NUL or PNL scheme are needed.

Numerically, MNL, NUL, and PNL are all (seemingly) legitimate approximations to the same governing equation (1) as discretized in the forward Euler form equation (2) (type-II uncertainty). They nevertheless represent different biogeochemical coupling between mineral nitrogen, plants, and microbes (type-I uncertainty). When the actual numerical representation of nitrogen limitation is not explicitly reported (which is common in the literature), one would regard the BGC models using these three schemes as structurally identical and numerically similar (and indeed for nitrogen unlimited conditions, these three approaches lead to identical model predictions (Tang and Riley, 2016)). However, because ecosystem carbon sequestration is the difference between several large magnitude nitrogen-limited ecosystem carbon fluxes, we hypothesize that different *numerical* nitrogen limitation schemes will lead to different predictions of ecosystem carbon dynamics.

We therefore in this study address two hypotheses:

(H1): The ambiguous numerical implementation of nitrogen limitation will lead to large uncertainty in simulated ecosystem carbon dynamics.

(H2): Uncertainty from the model time-step size is smaller than that resulting from the use of different nutrient limitation schemes.

We evaluated the above hypotheses using the ALMv0 model that integrates BeTR—a numerically robust reactive transport module (RTM) for biogeochemical transport and reactions (Tang et al., 2013)—with simulations of both historical
and future RCP4.5 emission scenario atmospheric CO₂ forcing. We compared our simulated uncertainty to that reported for the CMIP5 models (Shao et al., 2013), and assert H1 as true when these two sets of experiments are of comparable spread in terms of NEE (net ecosystem exchange of carbon). H2 is asserted as true if the site-level simulations indicate the time-step size affects the simulation less than using different numerical implementations as indicated in equations (3), (6) and (7).

Below we describe the model configurations and simulation protocols, present and discuss our model results, and finally give recommendations on how to remove or alleviate this new type of uncertainty (i.e., ambiguous numerical nutrient limitation).

2. Methods

2.1 Model configuration

We applied ALMv0-BeTR to explore how different numerical schemes of nitrogen limitation affect the predicted ecosystem uptake of atmospheric CO₂. BeTR is a multiphase RTM that consistently represents the transport (including multiphase diffusion, advection, ebullition, and gas phase arenchyma transport) for an arbitrary number of chemical tracers, which for this study includes seven carbon pools (Koven et al., 2013), and eight abiotic tracers, N₂, O₂, Ar, CH₄, CO₂, NH₄⁺, N₂O, and NO₃⁻. Compared to the first version of BeTR in CLM4 (Tang et al., 2013), ALMv0-BeTR improves the numerical treatment of dual phase diffusion (Tang and Riley, 2014) and advection (Manson and Wallis, 2000) (see Figure S1 for a demonstration of its numerical accuracy in tracer transport), and uses F90’s object oriented polymorphism to implement different BGC formulations within the same biophysical environment. As in the default ALMv0 BGC, which is the de facto CLM4.5BGC (Koven et al., 2013; Oleson et al., 2013), all BeTR BGC implementations do not physically transport NH₄⁺.

We implemented the biogeochemistry of all BeTR BGC models using the Peterson matrix based formulation (Russell, 2006; Tang and Riley, 2016), so that minimal modification was needed to implement the three nitrogen limitation schemes (i.e., MNL, NUL, and PNL).

We note that ALMv0 CLM4.5BGC uses an instantaneous, supply-demand ansatz (SDA; Parida, 2011) for downregulating GPP. Under nitrogen-limited conditions, this approach first calculates the ratio between existing soil mineral nitrogen pool and total potential nitrogen uptake (by plants and microbes) to avoid negative mineral nitrogen stock, and then multiplies this ratio with the nitrogen unlimited GPP to obtain the down-regulated GPP. This ansatz approach unrealistically assumes that root nutrient uptake instantaneously affects leaf photosynthesis and artificially restricts the plant and microbial nutrient competition to occur before plant carbon allocation. We recently showed that this approach (1) leads to very unrealistic diurnal GPP cycles (Ghimire et al. 2016) and (2) has not been corroborated by observations (Zhu et al., 2016), even though it may be ecologically convenient for analysing long-term ecosystem biogeochemistry with a time step of years. We therefore removed this down-regulation scheme in all BeTR BGC
simulations by adding a nitrogen storage pool to fulfil the nitrogen demand from GPP and refill the nitrogen storage pool through plant uptake (in presence of microbial competition).

**ALMv0-CLM4.5** BGC employs a fixed CN stoichiometry for plants and a CENTURY-like (Parton et al., 1988) formulation for soil BGC, where the latter represents microbial population dynamics and associated biogeochemical activity implicitly. All models used here allow plants and microbes to compete equally (or proportionally) for $\text{NH}_4^+$ and $\text{NO}_3^-$, and assume that both plants and organic matter decomposers assimilate $\text{NH}_4^+$ over $\text{NO}_3^-$. The first assumption (on whether the uptake of $\text{NH}_4^+$ and $\text{NO}_3^-$ is proportional to their pool sizes) is now under intense debate (e.g., Gerber et al., 2010; Zaehle and Friend, 2010; Thomas et al., 2015; Niu et al., 2016), whereas the second assumption is very likely unrealistic because (1) it restricts the model to execute nitrogen limitation after oxygen limitation (as $\text{NO}_3^-$ demand by denitrifiers is a function of oxygen and applying nitrogen limitation requires knowing the relative uptake demand of $\text{NH}_4^+$ over $\text{NO}_3^-$), even though they occur simultaneously in the real world and (2) a grid cell in any large scale BGC model actually represents the average of a heterogeneous soil, so the uptake of $\text{NO}_3^-$ should never be zero as long as some $\text{NO}_3^-$ exists.

To evaluate hypothesis (H1), we used five BGC model configurations implemented in **ALMv0-BeTR** (Table 1). Among them, the three BGC formulations (MNL, NUL, and PNL) differ in their numerical interpretations of nitrogen limitation. Since all model formulations in BeTR require identical model inputs, we also tested the model sensitivity to initial conditions by comparing PNL with PNLC, where the latter uses the code base of PNL and initial conditions from the NUL simulation. Simulations PNLC and NUL are compared to demonstrate the effect of different nitrogen limitation implementations with the same initial conditions. The final model configuration, PNLO, when compared to PNL, illustrates the ordering dependence of substrate limitation (for oxygen and nitrogen). To prevent the model PNLO from crashing (on negative values of oxygen or mineral nitrogen), we first predicted the relative demand for $\text{NH}_4^+$ and $\text{NO}_3^-$ based on total mineral nitrogen availability, then implemented oxygen limitation on nitrification and decomposition, and finally applied nitrogen limitation to microbes and plants a second time to obtain the corrected nitrogen uptake for plants and microbes. This requirement to apply nitrogen limitation in a predictor-corrector manner is not easily observable from the governing equations of the BGC model and demonstrates (1) that the default **ALMv0-CLM4.5** BGC model structures of plant-soil nitrogen interactions are problematic and (2) (once more) that numerical implementations of nutrient limitations in ESM land models may imply (sometimes unacknowledged) different ecological dynamics that is not described in the governing equations. We run our global simulations from 1850 to 2300 (see simulation protocol) and compare the output from 2006-2100 to the reported NEE for CMIP5 simulations (Shao et al., 2013) to evaluate H1.
The second hypothesis (H2) is evaluated with four example single gridcell simulations in geographically and climatically distinct locations (Figure 3): (74.67°W, 40.6°N; Eastern U.S.), (26.22°E, 67.7°N; Northern Finland), (50.02°W, 4.88°S; North East Brazil), and (51.5°W, 30.0°S; South Brazil). These gridcells were chosen to illustrate spatial heterogeneity in how time stepping strategies would influence simulated ecosystem carbon dynamics. We adopted the strategy from Tang and Riley (2016) (their appendix D) for adaptive time stepping and designated relevant simulations with PNL-adapt. H2 is evaluated by comparing the effect of adaptive-time stepping to that of using different numerical implementations of nitrogen limitation.

2.2 Simulation protocol

All model simulations were first run to preindustrial equilibrium using the spinup protocol in Koven et al. (2013) with the QIAN climate forcing data (cycled for 1948-1972; Qian et al., 2006). The model output by the end of spinup was then used for simulations in the contemporary period 1850-2000 with diagnostic atmospheric CO₂ concentrations. The RCP4.5 scenario atmospheric CO₂ concentrations (starting from 2006; see Figure S2b) were used together with the cycled QIAN climate forcing for the simulation period 2001-2300. We did not apply the climate anomaly representing future climate change to the RCP simulations; therefore the simulated carbon dynamics over 2001-2300 only represented the effects of changing atmospheric nitrogen deposition (Figure S2a), atmospheric CO₂ (Figure S2b), and land use change. We expect that including more uncertainty sources (such as uncertain future climate) will further strengthen the conclusions of our study (e.g. Tang and Zhuang, 2008). We finally note that the decision to run the simulations to 2300 is inspired by Randerson et al. (2015) and is just to push the models to one type of extreme and see if they would behave unexpectedly.

3 Results

3.1 Global simulations for the contemporary period 1850-2000

For the last decade (1991-2000) of the historical simulation period 1850-2000, the six model simulations gave very similar latitudinal distributions of several important variables (Figure 1). Small differences were found for latitudinal distributions of total soil organic carbon (Figure 1f), total soil organic nitrogen (Figure 1g), total vegetation carbon (Figure 1h) and total vegetation nitrogen (Figure 1i). Particularly for the July latent heat flux (Figure 1e), all simulations overlap, which is consistent with the relatively small differences in July leaf area index (LAI), GPP, NPP, and total vegetation carbon (Figure 1b, c, d and h) and that plant transpiration has a fast response to climate forcing (which is the same in all six model simulations). The overall close agreement between the default simulation (purple line) and all five BeTR-based simulations indicates that (1) the BeTR-based simulations are behaving in a reasonable way as compared to ALMv0 and (2) it requires a long time for the effect of different nitrogen limitation schemes to emerge in the simulations. This second observation is consistent with the usually high ecosystem carbon to nitrogen ratio and that ecosystem carbon stocks are cumulative.

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differences between the large fluxes of ecosystem carbon uptake (i.e., GPP) and ecosystem carbon loss (respiration and disturbances).

In contrast to the high degree of similarity between many of the variables simulated by the five BeTR-based models, the historical trajectories of cumulative NEE (positive means emitting CO$_2$ into the atmosphere) are very different (Figure 1a). Among the MNL, NUL, and PNL simulations, PNL (red line) had higher land carbon release compared to NUL (green line; an almost carbon neutral land by year 2000) and MNL (blue line; a cumulative land carbon uptake of about 40 Pg C by year 2000). The cumulative NEE simulated by PNLO (black line) is very similar to that by PNL, yet the ordering dependence still lowered the cumulative carbon release by about 50 Pg C by year 2000. PNLC (cyan line) showed an anomalously high release of land carbon resulting from enhanced decomposition of coarse woody debris (Figures S3-p1, p1, and p1), which is reflected in the higher heterotrophic respiration (Figure S4c) driven by more efficient decomposer nitrogen immobilization in PNL as compared to NUL. Although an in depth analysis will be provided using point simulations (section 3.3), this more efficient nitrogen uptake by PNL can be simply explained by observing the similar nitrogen input from fixation and deposition between the models (results are not shown but can be inferred from the almost overlapping NPP, which controls nitrogen fixation in this version of ALM), and (as shown mathematically in Supplemental Material) that the nitrogen uptake calculated from equation (6) (for NUL) is smaller than that from equation (7) (for PNL).

The huge difference between PNL and PNLC in the cumulative NEE (positive means emitting CO$_2$ into the atmosphere) are very different (Figure 1a). Among the five BeTR simulations, the RCP4.5 trajectories of cumulative NEE diverged into three groups (defined as the region between 23.2° S and 23.2° N) and NUL about 1.10 Pg C (~1400 g C m$^{-2}$) by year 2300; and PNLC predicted a small carbon loss of about −130 Pg C (~−4000 g C m$^{-2}$) by year 2300. The tropics (defined as the region between 23.2° S and 23.2° N) showed larger divergence (Figure 2b1) with a high carbon gain predicted by MNL about 1180 Pg C (~19000 g C m$^{-2}$) and NUL about 1150 Pg C (~18700 g C m$^{-2}$) by year 2300, and a lower carbon gain by PNL about 260 Pg C (~2800 g C m$^{-2}$) and PNLO about 230 Pg C (~2200 g C m$^{-2}$), and about −6 Pg C (~−100 g C m$^{-2}$) loss by PNLC. The divergence in the Arctic region (defined with latitudes north of 66.3° N; Figure 2c1) is smallest among the three regions, with a high carbon gain about 16 Pg C (~1400 g C m$^{-2}$ by MNL and NUL), a small carbon gain about 4 Pg C (~40 g C m$^{-2}$ by PNL and PNLO), and a large carbon loss about −14 Pg C (~−1200 g C m$^{-2}$) by year 2300.

3.2 Global simulations for the period 2001-2300

Although having very similar carbon and nitrogen stocks for the decade of 1991-2000 (Figure 1), the five BeTR simulations driven by the RCP4.5 atmospheric CO$_2$ concentrations diverged into three groups for 2001-2300 (Figure 2). For the north temperate region (i.e., north of 23.2° N and south of 66.3° N; Figure 2a1), simulations NUL and MNL almost overlapped and predicted a carbon gain of about 250 Pg C (~3200 g C m$^{-2}$) by year 2300; simulations PNLO and PNL almost overlapped and predicted a carbon gain of about 10 Pg C (~1400 g C m$^{-2}$) by year 2300; and PNLC predicted a small carbon loss of about −380 Pg C (~−4900 g C m$^{-2}$) by year 2300. The tropics (defined as the region between 23.2° S and 23.2° N) showed larger divergence (Figure 2b1) with a high carbon gain predicted by MNL about 1180 Pg C (~19000 g C m$^{-2}$) and NUL about 1150 Pg C (~18700 g C m$^{-2}$) by year 2300, and a lower carbon gain by PNL about 260 Pg C (~2800 g C m$^{-2}$) and PNLO about 230 Pg C (~2200 g C m$^{-2}$), and about −6 Pg C (~−100 g C m$^{-2}$) loss by PNLC. The divergence in the Arctic region (defined with latitudes north of 66.3° N; Figure 2c1) is smallest among the three regions, with a high carbon gain about 16 Pg C (~1400 g C m$^{-2}$ by MNL and NUL), a small carbon gain about 4 Pg C (~40 g C m$^{-2}$ by PNL and PNLO), and a large carbon loss about −14 Pg C (~−1200 g C m$^{-2}$) by year 2300.
While we do not place high confidence on the predicted numerical value (as discussed previously and below), the global terrestrial carbon stocks change (Figure S6) between 2006 and 2100 spreads from 10 Pg C (weak sink; PNLIC) to 680 Pg C (sink; MNL), which approximately encapsulates the range (22–456 Pg C) reported in Shao et al. (2013) (their Table 4) for the CMIP5 simulations. By 2300, the predicted global terrestrial carbon stock change ranges from a source of about −290 Pg C (PNLIC) to a carbon sink of about 1500 Pg C (with MNL being slightly higher than NUL). We note that this 1500 Pg C sink is close to a reduction of 700 ppmv atmospheric CO₂ which is greater than the 550 ppmv atmospheric CO₂ forcing. This clearly indicates that the BGC model structure of ALMv0 is questionable (and so we don’t place a good confidence on these numbers). Terrestrial carbon stock changes for the PNL and PNLO simulations fall between the predictions by PNLIC and MNL, with a carbon sink of 520 and 480 Pg C, respectively (Figure S6a). Since nitrogen limitation quantitatively increases across the model configurations (PNL < NUL < MNL), the sequential increases in carbon uptake (MNL > NUL > PNL) in response to the RCP4.5 atmospheric CO₂ trajectory imply that ALMv0 and CLM4.5BGC (which both use the MNL scheme) may predict too strong global CO₂ and nitrogen fertilization effects. We acknowledge that this stronger CO₂ fertilization effect resulting from stronger nitrogen limitation (as implied in the numerical implementations; see Supplemental Material) may first appear counter-intuitive, yet it can be reasonably explained through relevant ecological mechanisms (which lead to a stronger increase in heterotrophic respiration than in NPP under more efficient nitrogen uptake; see discussion in section 4.1). We also found that the predicted total soil carbon change is more sensitive than the total vegetation carbon change (Figure 2 and Figure S6) in response to the different nitrogen limitation implementations, indicating stronger nitrogen regulation of soil carbon stocks.

3.3 Point simulations for the four sites

For the group of simulations conducted at the four grid points (Figure 3), we observed similar divergences as those in the global simulations (Figure 2): the MNL scheme (blue lines) predicted higher carbon gain than did the PNL scheme (red lines), yet the NUL predictions (green lines) almost overlapped with those by MNL. Invoking adaptive time-stepping (PNL-adapt; magenta lines) further decreased the predicted carbon gain, which could be explained by the even more effective nitrogen uptake implied by the PNL scheme under smaller time steps. We also switched the computing order between reaction and transport for PNL-adapt (which like all simulations reported in this text calculates biogeochemical reaction before transport) and only found negligible difference (Figure S9).

3.4 Results of hypotheses evaluation

Taking all simulations together, we conclude that hypothesis H1 is affirmed given the spread of our simulated cumulative land carbon uptake is larger than that in Shao et al. (2013) for CMIP5 models. Meanwhile, H2 is satisfied in some, but not other, sites and that the size of the numerical time step could have either significant (Figure 3a) or secondary (Figure 3b, c, and d) importance on simulated ecosystem carbon stocks trajectories.
4. Discussion

Below we first discuss how the three different numerical implementations of nitrogen limitation led to different partitioning of nitrogen fluxes. Then we explore the importance of the coarse woody debris in affecting the simulated spread in land-atmosphere carbon exchange. Finally we give our recommendations on how to alleviate the type of uncertainty we identified in this study.

4.1 Reasons for the large C cycle differences between different nitrogen limitation implementations

We observed that PNL, NUL, and MNL schemes predict sequentially stronger nitrogen limitation under the same mineral nitrogen availability (Supplemental Material). For biogeochemical models like ALM that resolve mineral nitrogen into ammonium and nitrate (together with the assumed preference of ammonium over nitrate), this order of limitation translates into sequentially less effective plant and microbial assimilation of ammonium and stronger uptake of nitrate nitrogen. Indeed, we found PNL-adapt predicted the highest nitrification rate (as nitrifiers are competing for ammonium in ALM) followed by PNL and MNL (which overlapped with NUL; see Figure 4a1, b1, c1, a2, b2 and c2), leading to the same ranking of soil nitrate abundance (Figure S10) and nitrate loss through aqueous transport (Figure 4a4, b4, and c4). The difference in denitrification rates as simulated by different nitrogen limitation schemes is also evident, with the lowest value predicted by PNL-adapt, and increasing in MNL (which overlaps NUL) and then PNL. The simulations at 51.5° W, 30.0° S (which ALM identifies as a C3 grassland) only qualitatively resemble those at the other three sites, yet the ranking of soil nitrate abundance remains (Figure S10d). Corresponding to the nitrogen dynamics, the ecosystem heterotrophic respiration also increases in the order of MNL (which overlaps NUL), PNL, and then PNL-adapt, except for the period after 1980 for the fourth site (Figure 5d), indicating a strong sensitivity of carbon dynamics to nitrogen processes. For global simulations driven by RCP 4.5 atmospheric CO₂ forcing over 2001-2300, this stronger increase of respiration led PNL to predict about 3200 Pg C more respiration than did MNL, yet the NPP predicted by PNL only increased by 1900 Pg C as compared to that by MNL, which together led the PNL scheme to predict a lower CO₂ fertilization effect.

4.2 High sensitivity of coarse woody debris dynamics to nitrogen

We observed that the response of coarse woody debris (CWD) pool dominated the simulated difference in total land-atmosphere carbon exchange during both the contemporary period 1850-2000 (Figure S3) and the projection period 2001-2300 (Figures S7 and S8). A smaller fraction is contributed from carbon in woody product and seed (see second rows of Figures S3, S7 and S8). In ALMv0/CLM4.5BGC, coarse woody debris is accumulated from mortality due to fire (predicted with the model by Li et al. (2012a, b)) and background death (2% per year; Oleson et al., 2013), harvest and land use change, and it is lost through decomposition into lignin and cellulose. The decomposition of CWD immobilizes nitrogen, and is assumed to produce no CO₂, where the latter is obviously contradictory to reality (e.g. Gough et al., 2007).

Nevertheless, the more efficient nitrogen uptake as ranked in PNL, NUL and MNL (supplemental material) has led to the...
sequentially higher loss of CWD in the order of low to high as MNL, NUL, PNL (PNLO) and PNLIC (Figures S7 and S8). PNLIC has predicted the highest loss of CWD, because it uses the initial condition from NUL and NUL has accumulated more CWD during the spinup period due to its less efficient nitrogen uptake as compared to PNL. Regionally, the tropics showed the largest spread in the predicted CWD loss (~121–214 g C m⁻² yr⁻¹), followed by the north temperate region (~156–10 g C m⁻² yr⁻¹), south of 23° S (~10–8 g C m⁻² yr⁻¹) and the Arctic (~4–2 g C m⁻² yr⁻¹; see Figure S6 and S7). Such high sensitivity of the CWD dynamics with respect to the nitrogen dynamics further indicates urgency to develop robust implementations of nitrogen limitation in ESM land biogeochemical models.

4.3 Strategies for robust carbon and nitrogen coupling

Through the above results we show that ambiguous numerical implementation of nitrogen limitation could have led to a large carbon cycle prediction uncertainty. To rectify this situation, we have four recommendations (surrounding both the growth-controlling and growth-limiting processes) to help achieve a numerically robust coupling between carbon and nitrogen (or more generally nutrient) dynamics.

First and foremost, nutrient limitations should be handled automatically through a robust numerical solver (i.e. a better classification of the growth-limiting and growth-controlling processes), rather than being applied to individual processes through the convenient law of the minimum, an approach that has yet been challenged by observations (e.g. O’Neill, 1989; Danger et al., 2008), and is mechanistically redundant (appendix A). In reality, nutrient limitations emerge from continuous interactions among all entities and substrates in the ecosystem. Analytically applying law of the minimum to each of the modelled entities can turn the emergent limitation into a specific mechanism constraint that ignores interactions between competing entities. If a strategy is also employed to avoid (unphysical) negative numerical solutions (which is necessary), an unwanted double counting of substrate limitation will occur. Likewise, the numerical Monod-term based approach (e.g., Tang et al., 2016) incorrectly applies the nutrient limitation as an emergent constraint, as it introduces a specific mechanism constraint that depends on an ambiguously defined residual substrate concentration.

Second, we recommend models explicitly represent substrate kinetics for substrate competition between all consumers (i.e. better formulation of the growth-controlling process). On the one hand, substrate kinetics naturally have the property that as substrate concentrations decrease, uptake fluxes will smoothly decrease. On the other hand, unlike the numerical Monod term (in Tang et al., 2016; which can be equally replaced with functions like \( S^n/(S^n + K_s^n) \), where \( n \) is the quantitative order and \( K_s \) is the numerical half saturation constant), appropriately applied substrate kinetics, e.g., the Equilibrium Chemistry Approximation (ECA) kinetics (Tang and Riley, 2013; Tang, 2015; Zhu et al., 2016), have a mechanistic underpinning for the interactions between entities involved in substrate dynamics. In particular, the ECA kinetics allows for an explicit formulation of entity interactions for each substrate, whereas the application of Michaelis-Menten kinetics will render representation of competitive pressures into the system-wise numerical constraint, possibly...
causing inconsistencies between the conceptual model, its governing equations, and the numerical solution (see the litter
decomposition example in Tang and Riley (2013)).

Third, we contend that more robust numerical solvers should be employed to solve the BGC governing equations
(i.e. better implementation of the growth-limiting process). Terrestrial biogeochemical modelling has traditionally not paid
sufficient attention to this issue: model equations are often integrated with the single step Euler forward scheme (with a few
exceptions such as the TEM model (Raich et al., 1992) and the ED model (Knox, 2012), which used multi-step methods such
as the Runge-Kutta scheme), and ad hoc manipulations are invoked to rectify the unphysical numerical solutions (e.g., see
discussions in Tang and Riley (2016)). This may not be a severe issue when the models are of low complexity (e.g., the
CMIP5 models are mostly carbon-only models), where chances of unphysical solutions are less likely to occur. However,
there are urgent scientific reasons to introduce more mechanisms into terrestrial biogeochemical models (e.g., Wieder et al.,
2015b) for better and more comprehensive analyses of carbon-climate feedbacks. In particular, the migration from single-
layer to vertically resolved models is required to correctly simulate global soil carbon stocks, especially for Arctic
ecosystems (Koven et al., 2013, 2015b). For ecosystems such as peatlands, wetlands, rice paddies and tropical forests, the
soil physical environmental will frequently fluctuate between wet and dry conditions, causing strong shifts in soil redox
status. These dynamics will make the problem of substrate limitation more likely for different substrates over time. The first
order explicit reaction-based flux back tracing algorithm proposed in Tang and Riley (2016) is helpful to avoid unphysical
negative substrate concentrations during model execution and is numerically consistent with the processes represented in the
governing equations (thus it satisfies the Lax equivalence theorem (Lax and Richtmyer, 1956)). However, its explicit time
stepping approach may cause a ‘zigzag’ phenomena or premature convergence in some unusual cases (e.g., Figure 21). The
implicit scheme may also have strong time stepping dependence, and for complex biogeochemical systems, clipping and
variable transform may be needed for the implicit scheme to maintain positive solutions for concentrations (Tang et al.,
2016). However, as we discussed, the clipping approaches can introduce mass balance errors into the model. One possible
candidate to alleviate the time-stepping dependence is the exponential integrator (e.g. Tuckmantel, 2010), but it may still
suffer from violating the strict mass balance constraint that is guaranteed in Tang and Riley’s approach. We will present our
exercise of the exponential integrator elsewhere.

Finally, we suggest that biogeochemical models should provide more transparent methods description for users to
identify uncertainties (i.e. better documentation of the growth-controlling and growth-limiting processes), and then apply
approaches to robustly test model structural uncertainty. In reviewing the literature, we rarely found sufficient information
regarding how substrate limitation is numerically implemented in different models. Even when it is available, this
information is usually buried within lengthy derivations of the governing equations, making it difficult to determine to what
extent the numerical solutions are robust to the types of problems identified above. It is possible to organize a model’s
governing equations into a set of clearly stated differential and algebraic equations, and solve them by simply invoking
available numerical solvers. Such an approach will allow (1) a robust testing of how a model’s simulation depends on the
numerical solver and (2) for assessment of model structural uncertainty if multiple models (or model realizations) are solved
with the same robust numerical solver. Standardizing this component of land models could dramatically improve prediction uncertainty quantification and facilitate evaluation of new processes, leading to improved analysis of ecosystem dynamics and C-climate interactions.

5. Conclusions

The problems associated with ambiguous numerical implementation of substrate limitations are likely present in most ESM land models. Here, we used the coupled carbon-nitrogen dynamics in the version zero ACME land model as an example and demonstrated that the ambiguous numerical implementation of substrate limitation could be a serious type of carbon cycle uncertainty, comparable to the uncertainty across the suite of CMIP5 simulations. In particular, such uncertainty may imply the models are simulating (unacknowledged) ecological mechanisms that are inconsistent with the governing equations, which further lead to uncertainties with initial conditions, and ordering of model integrations. Given that more nutrient mechanisms will be introduced in the next generation of land biogeochemical models, this ambiguity will be even more important and potentially a very large source of uncertainty. For a robust numerical coupling of carbon and nutrient dynamics, we suggest modellers should: (1) abandon the law of minimum as an analytically explicit constraint to individual entities in the biogeochemical systems; (2) represent substrate competition in their models with explicit substrate kinetics, (3) use more advanced numerical solvers, and (4) document their model implementations with more technique details. With such, we could thence better understand if we are increasing the model complexity for the right reasons.

Appendix A. Example misuse of Liebig’s law of the minimum

We build our example based on the classic model by Leon and Tumpson (1975), which is

\[
\frac{dN_i}{dt} = N_i \left[ \min_j \left\{ \frac{g_j(R_j)}{q_{ij}} \right\} - D_j \right] \quad i = 1, \ldots, n \tag{A-1}
\]

\[
\frac{dR_j}{dt} = f_j(R_j) - \sum_i q_{ij} \min_j \left\{ \frac{g_j(R_j)}{q_{ij}} \right\} N_i \quad j = 1, \ldots, n \tag{A-2}
\]

where \(N_i\) is consumer \(i\) biomass density, \(R_j\) is resource \(j\) biomass density or concentration, or whatever variable is appropriate to the form of the resource. \(f_j(R_j)\) is net supply rate of resource \(j\), which could be either positive or negative. \(g_j(R_j)\) is rate of removal of the \(j\)-th resource by each individual of the \(i\)-th consumer population. \(q_{ij}\) is the conversion factor of units of \(j\) into units of \(i\) (or the reciprocal of substrate use efficiency of \(j\)-th substrate by \(i\)-th consumer population).
This model describes the growth of a community of populations (denoted by \( i \)) on a set of perfectly complementary substrates (denoted by \( j \)) based on Liebig's law of the minimum. However, this application of law of the minimum is incorrect. We back up our assertion with the following explanation.

Suppose there is only one population feeding on two perfectly complementary substrates, then by approximating equations (A-1) and (A-2) with the Euler forward form, we obtain

\[
N_i(t + \Delta t) = N_i(t) + \Delta t N_i(t) \left[ \min \left\{ \frac{g_{i1}(R_1(t))}{q_{i1}}, \frac{g_{i2}(R_2(t))}{q_{i2}} \right\} - D_i \right]
\]

(A-3)

\[
R_i(t + \Delta t) = R_i(t) + \Delta t f_i(R_i) - \Delta t q_{i1} \left[ \min \left\{ g_{i1}(R_1) \right\} \right] N_i, j = 1, 2
\]

(A-4)

\[
R_i(t + \Delta t) = R_i(t) + \Delta t f_i(R_i) - \Delta t q_{i2} \left[ \min \left\{ g_{i2}(R_2) \right\} \right] N_i, j = 1, 2
\]

(A-5)

Now suppose population \( N_i \) is locally limited by substrate \( R_1 \), such that \( g_{i1}(R_1(t))/q_{i1} < g_{i2}(R_2(t))/q_{i2} \), which leads to

\[
N_i(t + \Delta t) = N_i(t) + \Delta t N_i(t) \left[ \frac{g_{i1}(R_1(t))}{q_{i1}} - D_i \right]
\]

(A-6)

\[
R_i(t + \Delta t) = R_i(t) + \Delta t \left[ f_i(R_i) - g_{i1}(R_1) N_i \right]
\]

(A-7)

\[
R_i(t + \Delta t) = R_i(t) + \Delta t \left[ f_i(R_2) - \frac{q_{i2}}{q_{i1}} g_{i1}(R_1) N_i \right]
\]

(A-8)

Now define

\[
\lambda = \frac{g_{i1}(R_1(t)) q_{i2}}{g_{i2}(R_2(t)) q_{i1}}
\]

(A-9)

Where it can be verified that \( \lambda < 1 \). Then by entering equation (A-9) into (A-8), we obtain

\[
R_i(t + \Delta t) = R_i(t) + \Delta t \left[ f_i(R_2) - \lambda g_{i1}(R_1) N_i \right]
\]

(A-10)
Now if $R_1(t + \Delta t) > 0$ and $R_2(t + \Delta t) < 0$, both of which can be easily satisfied (note $f_2(R_2)$ could be negative), then population $N_1$ is de facto limited by substrate $R_2$, which is opposite to the “local constraint” that the growth of population $N_2$ is limited by substrate $R_1$. Now in order to avoid $R_2(t + \Delta t) < 0$, a numerical substrate limitation must be done, and the use of Liebig’s law of minimum in growth rate calculation in equation (A-3) is inappropriate such that it results in a double counting of substrate limitation. For a community of many populations and substrate, we expect such misuse of Liebig’s law of minimum could occur even more frequently, and should be avoided.

Author Contributions

J.Y. Tang designed the study and conducted the experiments. J.Y. Tang and W. J. Riley discussed the results and wrote the paper.

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The data used in this paper can be obtained by contacting the first author at jinyuntang@lbl.gov.
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Table 1. Model configurations used to evaluate the uncertainty of ambiguous numerical implementation of nutrient limitation.

<table>
<thead>
<tr>
<th>Simulation ID</th>
<th>Model configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNL</td>
<td>Mineral Nitrogen based Limitation scheme: only existing mineral nitrogen is available for uptake at current time step. It implements equation (3).</td>
</tr>
<tr>
<td>NUL</td>
<td>Net nitrogen Uptake based Limitation scheme: mineral nitrogen demand is calculated as the residual between total nitrogen demand and gross mineralization. It implements equation (6).</td>
</tr>
<tr>
<td>PNL</td>
<td>Proportional Nitrogen flux based limitation scheme: mineral nitrogen from gross mineralization and existing soil mineral nitrogen are competed equally by plants and microbes. It implements equation (7).</td>
</tr>
<tr>
<td>PNLIC</td>
<td>Like PNL, but it uses initial condition from NUL.</td>
</tr>
<tr>
<td>PNLO</td>
<td>Like PNL, but O₂ limitation comes after nitrogen limitation. However, a second nitrogen limitation required for avoiding model crash.</td>
</tr>
<tr>
<td>Default</td>
<td>ALMv0, which is the de facto CLM4.5BGC.</td>
</tr>
</tbody>
</table>
Figure 1. Model predictions for the contemporary period 1850-2000: (a) Cumulative net ecosystem exchange (NEE; positive into the atmosphere); (b) Gross primary productivity; (c) Net primary productivity; (d) July leaf area index; (e) July latent heat flux; (f) total organic soil carbon to 1 m depth; (g) total organic soil nitrogen to 1 m depth; (h) total vegetation carbon; and (i) total vegetation nitrogen. Results for (b)-(i) are averaged over 1991-2000.
Figure 2. Model simulations forced by the Representative Concentration Pathway 4.5 (RCP4.5) atmospheric CO₂ for year 2001-2300. Here total soil carbon includes litter carbon and soil organic matter as defined in CLM4.5 (Oleson et al., 2013); coarse woody debris is excluded (but can be found in Figure S7). All changes are calculated as relative to each of their initial carbon pool sizes at the start of the simulation (year 2000). The decadal oscillation shown in the figure is due to the cycling of the QIAN climate forcing.
Figure 3. Point simulations for the 4 specific grid cells using different model configurations. For each site, all simulations used identical initial conditions obtained from spinup with the PNL-adapt code. Note the color schemes are different from that in Figure 1 and Figure 2. The decadal oscillation shown in the figure is due to the cycling of the QIAN climate forcing.
Figure 4. Nitrogen fluxes for the four specific gridcell simulations obtained from different model configurations. The four columns from left to right correspond to the four locations specified in Figure 3. The decadal oscillation shown in the figure is due to the cycling of the QIAN climate forcing.
Figure 5. Heterotrophic respiration for the four specific gridcell simulations obtained from running different model configurations. The decadal oscillation shown in the figure is due to the cycling of the QIAN climate forcing.
Supplemental materials for “Potentially large uncertainty in ecosystem carbon dynamics resulting from ambiguous numerical coupling of carbon and nitrogen biogeochemistry: a demonstration with the ACME land model”

Jinyun Tang and William J. Riley

Earth and Environmental Science Area, Lawrence Berkeley National Lab (LBL), Berkeley, CA, United States

Corresponding to: J.Y. Tang, jinyuntang@lbl.gov

1. Introduction of contents

The first part of this supplemental material reports the proof of the sequentially weaker nitrogen limitation in the application of the MNL, NUL and PNL numerical nitrogen limitation schemes. The second part contains figures (S1-S11) that provide complementary information to support our results and conclusions in the main text.

2. Proof of the progressively weaker nitrogen limitation

We prove under the same soil mineral nitrogen availability and fluxes of $F_{S,\text{input}}$ and $F_{S,\text{uptake}}$ that the application of MNL, NUL and PNL schemes leads to progressively weaker nitrogen limitation.

We first prove $F_{\text{MNL,uptake}} < F_{\text{NUL,uptake}}$, where, without confusing the readers, the subscript $S$ was removed.

Because substrate $S$ is limited, $F_{\text{MNL,uptake}} < F_{\text{NUL,uptake}}$ is equivalent to

$$\frac{S(t)/\Delta t}{F_{S,\text{uptake}}} < \frac{S(t)/\Delta t}{F_{S,\text{uptake}} - F_{S,\text{input}}}$$

(S-1)
which is reduced to \( F_{S,\text{uptake}} - F_{S,\text{input}} < F_{S,\text{uptake}} \), a condition always holds.

We now prove \( \bar{F}_{\text{NUL,uptake}} < \bar{F}_{\text{PNL,uptake}} \). This requires

\[
\frac{S(t)/\Delta t}{F_{S,\text{uptake}} - F_{S,\text{input}}} < \frac{F_{S,\text{input}} + S(t)/\Delta t}{F_{S,\text{uptake}}} \tag{S-2}
\]

By rearranging the terms of (S-2), we have to show

\[
F_{S,\text{uptake}} S(t)/\Delta t < \left( F_{S,\text{input}} + S(t)/\Delta t \right) \left( F_{S,\text{uptake}} - F_{S,\text{input}} \right) \tag{S-3}
\]

which after some rearrangement becomes

\[
S(t)/\Delta t < \left( F_{S,\text{uptake}} - F_{S,\text{input}} \right) \tag{S-4}
\]

Since (S-4) is the definition of substrate limitation for the NUL scheme, it always holds under substrate limitation.

We now finish our proof.
List of supplemental figures

Figure S1. A demonstration of the tracer transport accuracy of BeTR. The Hydro water is water simulated with the biophysics module in the ACME land model. BeTR water is water tracer tracked in BeTR. Ideally, the linear fit should be one to one.
Figure S2: (a) Cumulative atmospheric deposition from 1850 to 2300. (b) Atmospheric CO₂ from 1850 to 2300. The small zigzag in (b) is due to that RCP4.5 CO₂ starts from 2006.
Figure S3: (a1), (b1) and (c1) are carbon changes in total coarse woody debris. (a2), (b2) and (c2) are changes in total product carbon and seed carbon.
Figure S4. Simulated cumulative carbon fluxes in the contemporary period 1850-2000.
Figure S5: Latitudinal distribution of simulated soil mineral nitrogen for 1991-2000. (a) Total soil mineral nitrogen; (b) $\text{NH}_4^+$ and (c) $\text{NO}_3^-$. 
Figure S6: Model simulations for the scenario Representative Concentration Pathway 4.5 (RCP4.5) atmospheric CO₂ for the years 2001-2300. Here total soil carbon includes litter carbon and soil organic matter as defined in CLM4.5; coarse woody debris is excluded. All changes are calculated as relative to each of their initial carbon pool sizes at the start of the simulation (i.e. end of year 2000). The oscillations as shown in the figure are due to the cycling of the QIAN climate forcing.
Figure S7: Simulated evolution of coarse woody debris carbon (a1-c1) and product and seed carbon (a2-c2) for the RCP 4.5 CO₂ driven period 2001-2300. These results are complementary to Figure 2 in the main text.
Figure S8: Simulated evolution of coarse woody debris carbon (a1-c1) and product and seed carbon (a2-c2) for the RCP 4.5 CO₂ driven period 2001-2300. These results are complementary to Figure S6 above.
Figure S9: Evaluation of the ordering effect for the point simulations. PNL-adapt-tr simulates transports ahead of biogeochemical calculations, whereas PNL-adapt does the opposite order. From left to right, the four columns are representing sites that are corresponding to the locations specified in Figure 3 of the main text.
Figure S10: Soil nitrate concentrations for the point simulations as obtained from different model configurations.
Figure S11: A demonstration of the zigzag phenomena and the strong time-stepping dependence of the numerical solution using Euler methods.

$x'=-15(x-0.05), \ x(0)=1$

- **Implicit Euler scheme**: $dt=1/8$
- **Implicit Euler scheme**: $dt=1/16$
- **Clipped Euler forward scheme**: $dt=1/8$
- **Clipped Euler forward scheme**: $dt=1/16$
- **True solution**